Financing and Investment of Foreign Subsidiary under International Tax Rate Differentials (Financial Modeling and Analysis)

Kijima, Masaaki; Tanaka, Keiichi; Tian, Yuan

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首都大学東京・社会科学研究科 木島 正明 (Masaaki Kijima) 田中 敬一 (Keiichi Tanaka)
Graduate School of Social Sciences
Tokyo Metropolitan University
京都大学・経済学研究科 田 園 (Yuan Tian)
Graduate School of Economics
Kyoto University

1 Introduction

With the advance of globalization process, it is more convenient and profitable for multinational companies (MNCs) to access foreign markets through foreign direct investment (FDI). FDI has been the subject of considerable research in the past decades. There are several FDI incentives: low corporate tax rates, cost reduction, financial subsidies provided by the foreign government, etc. Among these, international tax rate differentials are big concerns for MNCs. Hines (1999) and Grubert and Mutti (2000) document that the international differentials in corporate taxation significantly influence the location of FDI. Table 1 shows corporate tax rates in different countries in 2008.[1]

Table 1: Corporate tax rates in different countries (2008).

Country	Japan	US	Hong Kong	Ireland	global average
Corporate tax rate	40.7%	40%	16.5%	12.5%	25.9%

Source: KPMG's corporate tax rate survey 2008

To increase after-tax profits, MNCs have an incentive to shift profits from low- to high-tax countries. Since debt payments are deductible (tax benefits of debt), it is well known that international tax rate differentials create opportunities for debt shifting from high- to low-tax countries.[2] On the basis of a large sample of European firms over the 1994-2003 period, Huizinga *et al.* (2008) demonstrate that a foreign subsidiary's capital structure at its establishment is positively related to domestic corporate tax rate as well as the difference between the domestic and foreign tax rates. Moreover, they report that ignoring the international debt shifting arising from tax rate differentials would understate the impact of tax rate on debt policy by about 25%.

The objective of this paper is to develop a theoretic model to examine the impact of debt shifting on financing and investment decisions of foreign subsidiary, taking into consideration the foregn exchange (FX) rate uncertainty. Suppose a parent firm, which is located in domestic

[1] Dharmapala and Hines (2009) find that small and better-governed countries are more likely to become tax heavens.

[2] See Hines (1999), Mills and Newberry (2004), and Mintz and Smart (2004).

country with high tax rate, considers establishment of a subsidiary in foreign country with low tax rate. By investigating both the financing and investment decisions of the foreign subsidiary, we find that debt shifting induces earlier investment and default of foreign subsidiary, larger coupon level of debt, and higher leverage and credit spread. We demonstrate that when the difference between the domestic and foreign tax rates is large, the optimal leverage of foreign subsidiary at its establishment increases, and the inefficiency of investment due to ignoring debt shifting also increases. This is consistent with the empirical results in Huizinga *et al.* (2008). Moreover, we find that the investment of foreign subsidiary advances as FX rate uncertainty rises and becomes more correlated with the uncertainty in foreign market, which echoes the results in Goldberg and Kolstad (1995). The inefficiency of investment due to ignoring debt shifting decreases with the uncertainty of FX rate.

It is worth noting that so far the theoretical literature on FDI has mainly focused on either capital structure decision (see Panteghini (2009)) or investment decision (see Yu *et al.* (2007)), without considering the FX rate uncertainty. That is, uncertainty is modeled by a single process in the models above.

The main contribution of this paper is to enrich the analysis by introducing the FX rate uncertainty and examining the interaction of financing and investment decisions of foreign subsidiary. By doing so, we provide a theoretical framework that allows us to better interpret the empirical findings of FDI that incorporate FX rate and tax rate factors. Technically, this paper applies the first-hitting-time approach via change of measure to consider the two-dimensional problem. If we ignore FX rate dynamics, the discount rate differentials, and tax rate differentials, our model is reduced to the one-dimensional problem analyzed in Sundaresan and Wang (2007), which use a real options approach to examine financing and investment and capital structure decisions of a domestic firm.

The remainder of this paper is organized as follows. Section 2 describes the setup of the model. Section 3 examines the financing and investment decisions of the foreign subsidiary, employing the first-hitting-time approach via change of measure. Section 4 calibrates the model to analyze the characteristics of the solutions and provide several model predictions. Section 5 concludes.

2 Model setup

Suppose a parent firm, which is located in country d (domestic country), considers establishment of a subsidiary in country f (foreign country) at a fixed cost I in foreign currency. The parent firm and the foreign subsidiary together form a MNC, which is assumed to be risk neutral. The MNC can finance the irreversible investment cost I in foreign currency by issuing both equity and a perpetual debt with continuous coupon payment c. After establishing the foreign subsidiary at time T^i, the MNC instantaneously receives EBIT $X(t)$ in foreign currency, and pays coupon c in foreign currency to debtholders. We assume that both the EBIT $X(t)$ in foreign market and the FX rate $Q(t)$ (the domestic currency price of one unit of foreign currency) follow geometric

Brownian motions (GBMs) under probablity measure $\mathbb{Q}$:

$$\begin{aligned} \frac{\mathrm{d}X(t)}{X(t)} &= \mu_x \mathrm{d}t + \sigma_x \mathrm{d}w_1(t), & X(0) = x_0 > 0, \\ \frac{\mathrm{d}Q(t)}{Q(t)} &= (r_d - r_f)\mathrm{d}t + \sigma_q \left(\rho \mathrm{d}w_1(t) + \sqrt{1-\rho^2}\mathrm{d}w_2(t)\right), & Q(0) = q_0 > 0, \end{aligned} \tag{2.1}$$

where r_d and r_f are the risk-free interest rates in countries d and f, respectively, and $(w_1(t), w_2(t))$ is a two-dimensional standard Brownian motion under probablity measure $\mathbb{Q}$. That is, EBIT $X(t)$ is correlated to FX rate $Q(t)$ with a constant correlation coefficient ρ. This assumption reflects the well-known empirical evidence that there exists correlation between FX rate dynamics and equity market development. The initial value $X(0) = x_0$ is sufficiently low; i.e., EBIT in foreign market has not yet been favorable enough to cover the irreversible cost and the uncertainty.

For model simplicity, we follow Panteghini (2009) to assume that the parent firm produces a deterministic profit in domestic country with no default risk. A plausible explanation for the risk asymmetry in domestic and foreign countries is given by the fact that operating in the domestic country may be less risky than operating abroad. This is realistic since parent firms are aware of the characteristics of their own country, and thus can more easily predict and offset changes on their domestic business environment. Moreover, by assuming that the profit in domestic country is deterministic, the uncertainties are reduced from three dimensions to two dimensions (i.e., the foreign market uncertainty and the FX rate uncertainty). Then, the stopping times depend only on $X(t)(= X(t)Q(t)/Q(t))$ (see Eq.(2.6)). We emphasize that the main results and insights of this paper on financing and investment decisions of foreign subsidiary could hardly be obtained without the assumption above.[3]

Let τ_d and τ_f denote the corporate tax rates in countries d and f, respectively, with $\tau_d > \tau_f$. Without debt shifting, the MNC's instantaneous profit from foreign subsidiary (in domestic currency) at time t is

$$(1-\tau_f)(X(t)-c)Q(t). \tag{2.2}$$

Both the effective tax rate for EBIT in foreign market and the effective deductible tax-rate for coupon is τ_f.

However, if debt shifting is possible, the MNC can extract more tax benefits with debt shifting since coupon payments are tax deductible. Concretely, the MNC has an incentive to issue debt not locally in foreign country with low tax rate, but to issue debt in domestic country with high tax rate and shift debt to foreign subsidiary. Mills and Newberry (2004) report that although debt shifting may bring tax benefits, it is also associated with transaction costs. We assume that MNC shifts a precentage k of the foreign subsidiary's coupon c with a quadratic cost function $\nu(k) = \frac{n}{2}k^2$, $k \in [0,1]$, where $n \geq 0$ measures how costly for debt shifting. It can be easily checked that $\nu'(k) \geq 0$, $\nu''(k) \geq 0$, $\nu(0) = 0$. Then, with debt shifting, the MNC's

[3] Hu and Øksendal (1998) analyze the investment decision in an all-equity financing framework (i.e., without financing decision) and suggest the shape of the stopping region is linear when the parameters satisfy certain conditions in general n-dimensions.

instantaneous profit from foreign subsidiary (in domestic currency) at time t is

$$(1-\tau_f)\,(X(t)-c+kc)\,Q(t)-(1-\tau_d)(k+\nu(k))cQ(t).$$

Grouping terms yields

$$(1-\tau_f)X(t)Q(t)-(1-\tilde{\tau})cQ(t), \tag{2.3}$$

where

$$\tilde{\tau}=\tau_f+\phi(k)=\tau_f+(\tau_d-\tau_f)k-(1-\tau_d)\nu(k) \tag{2.4}$$

Note that while the effective tax rate for EBIT in foreign market is τ_f, the effective deductible tax-rate for coupon is $\tilde{\tau}$.

The optimal percentage k^* of coupon shifting can be obtained in the following:

$$k^*=\underset{k}{\operatorname{argmax}}\,\phi(k)=\begin{cases}1, & 0\le n\le\bar{n},\\ \frac{\bar{n}}{n}, & n\ge\bar{n},\end{cases} \tag{2.5}$$

where

$$\bar{n}=\frac{\tau_d-\tau_f}{1-\tau_d}.$$

That is, the optimal percentage k^* increases with the difference between the domestic and foreign tax rates. Substituting Eq.(2.5) into Eq.(2.4), we obtain $\tilde{\tau}$ as:

$$\tilde{\tau}=\begin{cases}\tau_d-\frac{1}{2}(1-\tau_d)n\in[\frac{1}{2}(\tau_d+\tau_f),\tau_d], & 0\le n\le\bar{n},\\ \tau_f+\frac{(\tau_d-\tau_f)^2}{2(1-\tau_d)n}\quad\in[\tau_f,\frac{1}{2}(\tau_d+\tau_f)], & n\ge\bar{n}.\end{cases}$$

Note that $\tilde{\tau}>\tau_f$, which implies that the effective deductible-tax rate is larger with debt shifting.

Although issuing debt can obtain tax benefits, it is also accompanied with default costs. As in Leland (1994), we consider a stock-based definition of default whereby equityholders inject funds in the subsidiary as long as equity value of the subsidiary is positive. In other words, equityholders default on their debt obligations the first time equity value of the subsidiary is equal to zero. We assume that the debt of the subsidiary is not guaranteed legally by the parent firm, because they are separate entities. Let T^b denote the time the foreign subsidiary goes into default, which is determined by the MNC. At the default threshold, $\alpha\in(0,1)$ part of the subsidiary's firm value is lost as default cost, the remaining $(1-\alpha)$ part belongs to debtholders of the subsidiary. The parent firm exists as a domestic firm after that.

Notice that the MNC's instantaneous profit in relation to foreign subsidiary, which is described in Eq.(2.3), is first-order homogeneous in $(X(t)Q(t),Q(t))$. According to McDonald and Siegel (1986), we define the stopping times T^i and T^b as follows:

$$\begin{aligned}T^i&=\inf\left\{t\ge 0,\ \frac{X(t)Q(t)}{Q(t)}\ge x^i\right\}=\inf\{t\ge 0,\ X(t)\ge x^i\},\\ T^b&=\inf\left\{t\ge T^i,\ \frac{X(t)Q(t)}{Q(t)}\le x^b\right\}=\inf\{t\ge T^i,\ X(t)\le x^b\}.\end{aligned} \tag{2.6}$$

3 Financing and Investment decisions of foreign subsidiary

Through this paer, we assume that equityholders make the decisions. In our model, there are two types of interrelated decisions in relation to foreign subsidiary: financing and investment decisions. The investment decision is characterized by an endogenously determined threshold; when the EBIT process of the foreign country $(X(t))_{t>0}$ reaches investment threshold x^i, MNC establishes the foreign subsidiary. The financing decision involves the choice of debt level and an endogenous default threshold of the foreign subsidiary. The coupon level of debt $c(x^i)$, which is characterized by a trade-off between the tax benefits and default costs of debt financing, is determined simultaneously with the investment decision of the foreign subsidiary. In contrast, the default threshold $x^b(c)$, which depends on coupon level, is determined after the foreign subsidiary is established. Note that the three endogenous variables in our model (i.e., x^i, $c(x^i)$, and $x^b(c)$) form a nested structure, and therefore enables us to examine the interaction between financing and investment decisions.

We derive the MNC's decisions using backward induction. Section 3.1 examines the default threshold of foreign subsidiary from the values after investment of foreign subsidiary. Section 3.2 analyzes the the coupon level of debt and optimal investment policy of the foreign subsidiary.

3.1 Default decision

To determine the bankrputcy threshold, we need first derive the values after investment of foreign subsidiary. According to our setup, for $T^i \leq t \leq T^b$, the equity value and debt value of foreign subsidiary (in domestic currency) are evaluated as

$$\begin{aligned} E(x,q) =& \mathbb{E}\left[\int_t^{T^b} \mathrm{e}^{-r_d(s-t)}\left[(1-\tau_f)X(s)Q(s)-(1-\tilde{\tau})cQ(s)\right]\mathrm{d}s\Big|(X(t),Q(t))=(x,q)\right], \\ D(x,q) =& \mathbb{E}\left[\int_t^{T^b} \mathrm{e}^{-r_d(s-t)}cQ(s)\mathrm{d}s\Big|(X(t),Q(t))=(x,q)\right] \\ &+\mathbb{E}\left[\mathrm{e}^{-r_d(T^b-t)}(1-\alpha)\frac{(1-\tau_f)X(T^b)Q(T^b)}{r_f-\mu_x-\rho\sigma_q\sigma_x}\Big|(X(t),Q(t))=(x,q)\right], \end{aligned} \tag{3.1}$$

respectively, where $\mathbb{E}[\cdot|(X(t),Q(t))=(x,q)]$ denotes the expectation operator under probability measure $\mathbb{Q}$, given that $(X(t),Q(t))=(x,q)$. We assume $r_f > \mu_x+\rho\sigma_q\sigma_x$ for convergency, where the term $\rho\sigma_q\sigma_x$ exists because of the correlation between EBIT and FX rate. The firm value is the sum of the equity value and debt value.

$$\begin{aligned} V(x,q) =& \mathbb{E}\left[\int_t^{T^b} \mathrm{e}^{-r_d(s-t)}\left[(1-\tau_f)X(s)Q(s)+\tilde{\tau}cQ(s)\right]\mathrm{d}s\Big|(X(t),Q(t))=(x,q)\right] \\ &+\mathbb{E}\left[\mathrm{e}^{-r_d(T^b-t)}(1-\alpha)\frac{(1-\tau_f)X(T^b)Q(T^b)}{r_f-\mu_x-\rho\sigma_q\sigma_x}\Big|(X(t),Q(t))=(x,q)\right]. \end{aligned} \tag{3.2}$$

For convenience, we consider the following process:

$$
\begin{aligned}
\mathrm{d}\ln X(t) &= \left(\mu_x - \frac{\sigma_x^2}{2}\right)\mathrm{d}t + \sigma_x \mathrm{d}w_1(t),\\
\mathrm{d}\ln Q(t) &= \left(r_d - r_f - \frac{\sigma_q^2}{2}\right)\mathrm{d}t + \sigma_q\left(\rho \mathrm{d}w_1(t) + \sqrt{1-\rho^2}\mathrm{d}w_2(t)\right),\\
\mathrm{d}\ln(XQ)(t) &= \left(\mu_x + r_d - r_f - \frac{\sigma_x^2 + \sigma_q^2}{2}\right)\mathrm{d}t + (\sigma_x + \rho\sigma_q)\mathrm{d}w_1(t) + \sqrt{1-\rho^2}\sigma_q \mathrm{d}w_2(t).
\end{aligned}
\tag{3.3}
$$

Then, the stopping times defined in Eq.(2.6) can be rewritten as

$$
T^i = \inf\{t \geq 0,\ \ln X(t) \geq \ln x^i\}, \quad T^b = \inf\{t \geq T^i,\ \ln X(t) \leq \ln x^b\}. \tag{3.4}
$$

The equity value and debt value in Eq.(3.1) can also be written as

$$
\begin{aligned}
E(x,q) &= \mathbb{E}\left[\int_t^{T^b} \mathrm{e}^{-r_d(s-t)}\left[(1-\tau_f)\mathrm{e}^{\ln(XQ)(s)} - (1-\tilde{\tau})c\mathrm{e}^{\ln Q(s)}\right]\mathrm{d}s \middle| (X(t),Q(t)) = (x,q)\right],\\
D(x,q) &= \mathbb{E}\left[\int_t^{T^b} \mathrm{e}^{-r_d(s-t)}c\mathrm{e}^{\ln Q(s)}\mathrm{d}s \middle| (X(t),Q(t)) = (x,q)\right]\\
&\quad + \mathbb{E}\left[\mathrm{e}^{-r_d(T^b-t)}(1-\alpha)\frac{(1-\tau_f)\mathrm{e}^{\ln(XQ)(T^b)}}{r_f - \mu_x - \rho\sigma_q\sigma_x} \middle| (X(t),Q(t)) = (x,q)\right],
\end{aligned}
\tag{3.5}
$$

Now, define a new probability measure $\mathbb{P}$ by

$$
\left.\frac{\mathrm{d}\mathbb{P}}{\mathrm{d}\mathbb{Q}}\right|_{\mathcal{F}_t} = \eta(t) = \mathrm{e}^{-v_z w_1(t) - \frac{1}{2}v_z^2 t},
$$

where $v_z = (\mu_x - \sigma_x^2/2)/\sigma_x$. Then (W_1, W_2) forms a two-dimensional standard Brownian motion.[4]

$$
\begin{aligned}
\mathrm{d}W_1(t) &= \mathrm{d}w_1(t) + v_z \mathrm{d}t,\\
\mathrm{d}W_2(t) &= \mathrm{d}w_2(t).
\end{aligned}
$$

Under the new probability mesure $\mathbb{P}$,

$$
\begin{aligned}
\mathrm{d}\ln X(t) &= \sigma_x \mathrm{d}W_1(t),\\
\mathrm{d}\ln Q(t) &= \left(r_d - r_f - \frac{\sigma_q^2}{2} - \rho\sigma_q v_z\right)\mathrm{d}t + \sigma_q\left(\rho \mathrm{d}W_1(t) + \sqrt{1-\rho^2}\mathrm{d}W_2(t)\right),\\
\mathrm{d}\ln(XQ)(t) &= \left(r_d - r_f - \frac{\sigma_q^2}{2} - \rho\sigma_q v_z\right)\mathrm{d}t + (\rho\sigma_q + \sigma_x)\mathrm{d}W_1(t) + \sqrt{1-\rho^2}\sigma_q \mathrm{d}W_2(t).
\end{aligned}
$$

The stopping times can be rewritten as

$$
\begin{aligned}
T^i &= \inf\{t \geq 0,\ W_1(t) = i\}, \quad i = \frac{\ln x^i - \ln x}{\sigma_x},\\
T^b &= \inf\{t \geq T^i,\ W_1(t) = b\}, \quad b = \frac{\ln x^b - \ln x}{\sigma_x}.
\end{aligned}
\tag{3.6}
$$

[4] We use the change-of-measure technique here as in Kijima *et al.* (2010).

By using the Laplace transform of first hitting time, we obtain the following proposition. The proof can be given upon request.

Proposition 3.1 (Values after investment of foreign subsidiary)
For $T^i \leq t \leq T^b$,
(i) equity value:

$$E(x,q) = \frac{1-\tau_f}{r_f - \mu_x - \rho\sigma_q\sigma_x}\left[xq - x^b q\left(\frac{x}{x^b}\right)^{\beta_1}\right] - \frac{1-\tilde{\tau}}{r_f} cq\left[1 - \left(\frac{x}{x^b}\right)^{\beta_2}\right]. \tag{3.7}$$

(ii) debt value:

$$D(x,q) = \frac{cq}{r_f}\left[1 - \left(\frac{x}{x^b}\right)^{\beta_2}\right] + (1-\alpha)\frac{1-\tau_f}{r - \mu_x - \rho\sigma_q\sigma_x} x^b q\left(\frac{x}{x^b}\right)^{\beta_1}. \tag{3.8}$$

(iii) firm value:

$$V(x,q) = \frac{1-\tau_f}{r_f - \mu_x - \rho\sigma_q\sigma_x}\left[xq - \alpha x^b q\left(\frac{x}{x^b}\right)^{\beta_1}\right] + \frac{\tilde{\tau}}{r_f} cq\left[1 - \left(\frac{x}{x^b}\right)^{\beta_2}\right], \tag{3.9}$$

where

$$\begin{aligned}
\beta_1 &= -\frac{v_z + \rho\sigma_q + \sqrt{K' + 2r_d}}{\sigma_x} < 0,\\
\beta_2 &= -\frac{v_z + \rho\sigma_q + \sqrt{K + 2r_d}}{\sigma_x} < 0,\\
K &= 2(r_f - r_d) + (v_z + \rho\sigma_q)^2,\\
K' &= 2(r_f - r_d - \mu_x - \rho\sigma_q\sigma_x) + (v_z + \sigma_x + \rho\sigma_q)^2.
\end{aligned}$$

The equity value of foreign subsidiary after investment (in domesitc currency) has two components: (i) the present value of EBIT without default; (ii) the present value of the coupon payments paid to the debtholders without default. Note that the coefficients of default option $\beta_1 \neq \beta_2$ generally, because the first hitting time of the processes $X(t)Q(t)$ and $Q(t)$ differs. The debt value of foreign subsidiary after investment (in domestic currency) also has two components: (i) the present value of coupon payments without default; (ii) the remaining firm value upon default. The firm value $V(x,q)$ is the sum of equity value and debt value.

The optimal default threshold of foreign subsidiary is determined to maximize the *ex post* equity value after debt has been issued, as in Leland (1994).

$$\left.\frac{\mathrm{d}E(x,q)}{\mathrm{d}x}\right|_{x=x^b} = 0.$$

Solving the equation above, we get

$$x^b = Ac, \tag{3.10}$$

where

$$A = \frac{\beta_2}{\beta_1 - 1}\frac{1-\tilde{\tau}}{1-\tau_f}\frac{r_f - \mu_x - \rho\sigma_q\sigma_x}{r_f}. \tag{3.11}$$

Note that the default threshold x^b is a linear function of the coupon level c.

3.2 Coupon level and investment decisions

Before turning to the analysis of coupon level and investment decisions, it is important to make a clear distinction between the *ex ante* equity value and the *ex post* equity value. While the *ex post* equity value is given by the present value of the cash flow accruing to equityholders after debt has been issued (see Eq.(3.7)), the *ex ante* equity value is given by the sum of the *ex post* equity value and debt value (see Eq.(3.9)) at the time it is issued. As a result, although equityholders choose the default threshold to maximize the *ex post* equity value, they choose the coupon level and investment threshold to maximize the firm value (*ex ante* equity value), internalizing both the tax benefits and default costs of debt financing. That is,

$$c^*(x^i) = \operatorname*{argmax}_c V(x^i, Q(T^i)), \tag{3.12}$$

Substituting $x^b = Ac$ into Eq.(3.9) with $(x, q) = (x^i, Q(T^i))$, the firm value upon investment can be expressed as:

$$V(x^i, Q(T^i)) = \frac{(1-\tau_f)x^i Q(T^i)}{r_f - \mu_x - \rho\sigma_q\sigma_x} + \frac{\tilde{\tau}}{r_f}\frac{x^i}{A}\frac{Ac}{x^i}Q(T^i)\left[1 - \left(\frac{Ac}{x^i}\right)^{-\beta_2} - \alpha\frac{\beta_2}{\beta_1 - 1}\frac{1-\tilde{\tau}}{\tilde{\tau}}\left(\frac{Ac}{x^i}\right)^{-\beta_1}\right]. \tag{3.13}$$

Therefore,

$$c^*(x^i) = \frac{x^i}{A}\operatorname*{argmax}_{m>0} f(m), \tag{3.14}$$

where

$$m = \frac{Ac}{x^i} = \frac{x^b}{x^i},$$

and

$$f(m) = m - m^{1-\beta_2} - \alpha\frac{\beta_2}{\beta_1 - 1}\frac{1-\tilde{\tau}}{\tilde{\tau}}m^{1-\beta_1}.$$

Substituting $Ac/x^i = m$ into Eq.(3.13), the firm value upon investment can be rewritten as

$$V(x^i, Q(T^i)) = B\frac{1-\tau_f}{r_f - \mu_x - \rho\sigma_q\sigma_x}x^i Q(T^i),$$

where

$$B(m) = 1 - \alpha m^{1-\beta_1} + \frac{\tilde{\tau}}{1-\tilde{\tau}}\frac{\beta_1 - 1}{\beta_2}m(1 - m^{-\beta_2}).$$

Having derived the firm value, we next analyze the optimal investment threshold. Since the investment cost financed by equity is $IQ(T^i) - D(x^i, Q(T^i))$, the equityholders choose the optimal investment threshold of foreign subsidiary as follows:

$$x^{i*} = \operatorname*{argmax}_{x^i} \mathbb{E}\left[e^{-r_d T^i}\left[E(x^i, Q(T^i)) - (IQ(T^i) - D(x^i, Q(T^i)))\right]\Big|(X(0), Q(0)) = (x_0, q_0)\right] \tag{3.15}$$

Note that the objective function of the maximization problem (3.15) is exactly the *ex ante* firm value of foreign subsidiary:

$$\begin{aligned} V^o(x_0,q_0;x^i) &= \mathbb{E}\left[\mathrm{e}^{-r_dT^i}\left[V(x^i,Q(T^i)) - IQ(T^i)\right]\Big|(X(0),Q(0)) = (x_0,q_0)\right] \\ &= \left(B\frac{1-\tau_f}{r_f-\mu_x-\rho\sigma_q\sigma_x}x^i - I\right)\mathbb{E}_0\left[e^{-r_dT^i}Q(T^i)\right]. \end{aligned} \tag{3.16}$$

Therefore, we choose the optimal investment threshold x^{i*} to maximize the *ex ante* firm value. Solving the maximization problem, and substituting x^{i*} into Eq.(3.14) and then Eq.(3.10), we finally obtain Proposition 3.2.

Proposition 3.2 (Optimal threshold and coupon level of foreign subsidiary)
The optimal investment threshold of foreign subsidiary is given by

$$x^{i*} = \frac{r_f-\mu_x-\rho\sigma_q\sigma_x}{1-\tau_f}\frac{\beta_3}{\beta_3-1}\frac{1}{B(m^*)}I, \tag{3.17}$$

where

$$\beta_3 = -\frac{v_z+\rho\sigma_q-\sqrt{K+2r_d}}{\sigma_x} > 1,$$

$B(\cdot)$ *is defined by Eq.*(3.15), *and* m^* *satisfies*

$$1-(1-\beta_2)(m^*)^{-\beta_2} + \alpha\beta_2\frac{1-\tilde{\tau}}{\tilde{\tau}}(m^*)^{-\beta_1} = 0. \tag{3.18}$$

The default threshold of foreign subsidiary is obtained as

$$x^{b*} = m^*x^{i*} = \frac{r_f-\mu_x-\rho\sigma_q\sigma_x}{1-\tau_f}\frac{\beta_3}{\beta_3-1}\frac{m^*}{B(m^*)}I. \tag{3.19}$$

The coupon level of foreign subsidiary is given by

$$c^* = \frac{x^{i*}}{A}m^* = \frac{\beta_3}{\beta_3-1}\frac{\beta_2}{\beta_1-1}\frac{r_f}{1-\tilde{\tau}}\frac{m^*}{B(m^*)}I. \tag{3.20}$$

The leverage and credit spread of foreign subsidiary upon investment are

$$\begin{aligned} L^*(x^{i*},Q(T^{i*})) &= \frac{D(x^{i*},Q(T^{i*}))}{V(x^{i*},Q(T^{i*}))} = \left[\frac{\beta_1-1}{\beta_2}\frac{m^*}{1-\tilde{\tau}}(1-(m^*)^{-\beta_2}) + (1-\alpha)(m^*)^{1-\beta_1}\right]\frac{1}{B(m^*)}, \\ CS^*(x^{i*},Q(T^{i*})) &= \frac{c^*Q(T^{i*})}{D(x^{i*},Q(T^{i*}))} - r_d = \left[1-m^{-\beta_2} + (1-\alpha)(1-\tilde{\tau})\frac{\beta_2}{\beta_1-1}m^{-\beta_1}\right]^{-1} r_f - r_d. \end{aligned} \tag{3.21}$$

The investment threshold, the coupon level, and the default threshold are all proportional to the investment cost I, due to the GBM assuption for the EBIT and FX rate processes Eq.(2.1). The ratio of default threshold to investment threshold is m^*, which is constant and determined by Eq.(3.18). Both the leverage and credit spread upon investment are constant and independent of investment cost I.

3.3 Special case

If ignoring the FX rate and the interest rate diffrentials (i.e., $\sigma_q = 0,\ \rho = 1,\ q_0 = 1,\ r_f = r_d$), then simple calculations give that

$$\beta_1 = \beta_2 = \gamma, \quad \beta_3 = \beta, \quad m^* = h(\tilde{\tau}), \quad B(m^*) = g(\tilde{\tau}), \tag{3.22}$$

where

$$\begin{aligned}
\beta &= -\frac{1}{\sigma_x^2}\left[\left(\mu_x - \frac{1}{2}\sigma_x^2\right) - \sqrt{\left(\mu_x - \frac{1}{2}\sigma_x^2\right)^2 + 2r_f\sigma_x^2}\right], \\
\gamma &= -\frac{1}{\sigma_x^2}\left[\left(\mu_x - \frac{1}{2}\sigma_x^2\right) + \sqrt{\left(\mu_x - \frac{1}{2}\sigma_x^2\right)^2 + 2r_f\sigma_x^2}\right], \\
h(\tilde{\tau}) &= \left[1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tilde{\tau}}\right)\right]^{\frac{1}{\gamma}} < 1, \\
g(\tilde{\tau}) &= 1 + \frac{\tilde{\tau}}{1-\tilde{\tau}} h(\tilde{\tau}) > 1.
\end{aligned} \tag{3.23}$$

Therefore, we obtain the following analytical results.

Lemma 3.1 (Special case)
If ignoring the FX rate and the interest rate diffrentials, the optimal investment threshold of foreign subsidiary is

$$x^{i**} = \frac{\beta}{\beta - 1}\frac{r_f - \mu_x}{1 - \tau_f}\frac{1}{g(\tilde{\tau})} I.$$

The default threshold of foreign subsidiary is obtained as

$$x^{b**} = h(\tilde{\tau}) x^{i**} = \frac{\beta}{\beta - 1}\frac{r_f - \mu_x}{1 - \tau_f}\frac{h(\tilde{\tau})}{g(\tilde{\tau})} I.$$

The coupon level of foreign subsidiary is given by

$$c^{**}(x^{i**}) = \frac{\gamma - 1}{\gamma}\frac{1 - \tau_f}{1 - \tilde{\tau}}\frac{r_f}{r_f - \mu_x} h(\tilde{\tau}) x^{i**} = \frac{\beta}{\beta - 1}\frac{\gamma - 1}{\gamma}\frac{r_f}{1 - \tilde{\tau}}\frac{g(\tilde{\tau})}{h(\tilde{\tau})} I.$$

The leverage and credit spread of foreign subsidiary upon investment are

$$\begin{aligned}
L^{**}(x^{i**}) &= \frac{\gamma - 1}{\gamma}\frac{1 - \kappa(\tilde{\tau})}{1 - \tilde{\tau}}\frac{h(\tilde{\tau})}{g(\tilde{\tau})}, \\
CS^{**}(x^{i**}) &= \frac{\xi(\tilde{\tau})}{1 - \xi(\tilde{\tau})} r_f,
\end{aligned}$$

where

$$\xi(\tilde{\tau}) = \left[1 - (1-\alpha)(1-\tilde{\tau})\frac{\gamma}{\gamma - 1}\right](h(\tilde{\tau}))^{-\gamma} \in (0, 1).$$

Moreover, if there is no debt shifting (i.e., $\tilde{\tau} = \tau_f$), the results above are reduced to those derived in Sundaresan and Wang (2007a).

4 Model implications

In this section, we calibrate the model to analyze the characteristics of the solutions and provide several model predictions. The basic parameter values are set as follows: $\mu_x = 0.01$, $\sigma_x = 0.35$, $\sigma_q = 0.1$, $\rho = 0.5$, $r_d = 0.04$, $r_f = 0.06$, $\tau_d = 0.4$, $\tau_f = 0.15$, $\alpha = 0.4$, $n = 0$, $I = 10$, $x = 1$, $q = 12$.

4.1 Inefficiency

Figure 1 plots the *ex ante* firm value and investment threshold of foreign subsidiary, where "N" and "S" denote no debt shifting and debt shifting, respectively. We find that the *ex ante* firm value of foreign subsidiary is larger and the investment of foreign subsidiary occurs earlier with debt shifting. Next, we consider the inefficiency due to ignoring debt shifting. The inefficiency

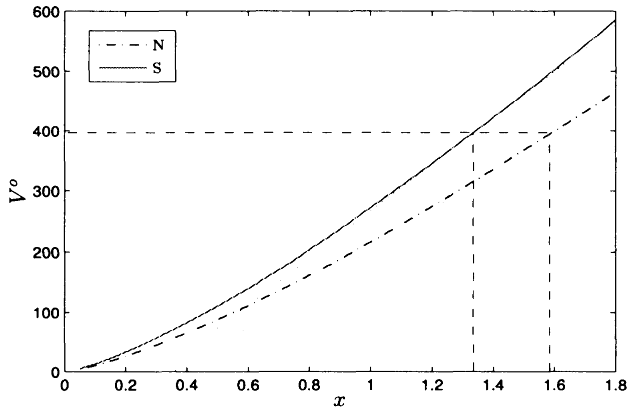

Figure 1: *Ex ante* firm value and investment threshold of foreign subsidiary.

in *ex ante* firm value and leverage are defined as follows:

$$\Delta V^{o*}(x_0, q_0; x^{i*}) = 1 - \frac{V_{\mathrm{N}}^{o*}(x_0, q_0; x_{\mathrm{N}}^{i*})}{V_{\mathrm{S}}^{o*}(x_0, q_0; x_{\mathrm{S}}^{i*})}, \quad \Delta L^*(x^{i*}, Q(T^{i*})) = 1 - \frac{L^*(x_{\mathrm{N}}^{i*}, Q(T_{\mathrm{N}}^{i*}))}{L^*(x_{\mathrm{S}}^{i*}, Q(T_{\mathrm{S}}^{i*}))}. \tag{4.1}$$

Figure 2 indicates that inefficiency of investment increases when tax rate differentials increase and FX rate uncertainty decreases. For our basic parameter values ($\Delta\tau = 0.25$, $\sigma_q = 0.1$), the ineffiecinecy in *ex ante* firm value is about 20%, and the ineffiecinecy in leverage is over 40%. Therefore, the impact of debt shifting cannot be ignored. This result echoes the empirical evidence in Huizinga *et al.* (2008), who report that ignoring debt shifting would understate the impact of tax rate on debt policy by about 25%.

4.2 Comparative statics of thresholds

Figure 3 demonstrates that the investment threshold decreases when (i) tax rate differentials increase (top left panel), (ii) debt shifting cost decreases (top right panel), (iii) the trend of FX rate rises (middle left panel), (iv) the uncertainty of FX rate rises (middle right panel),

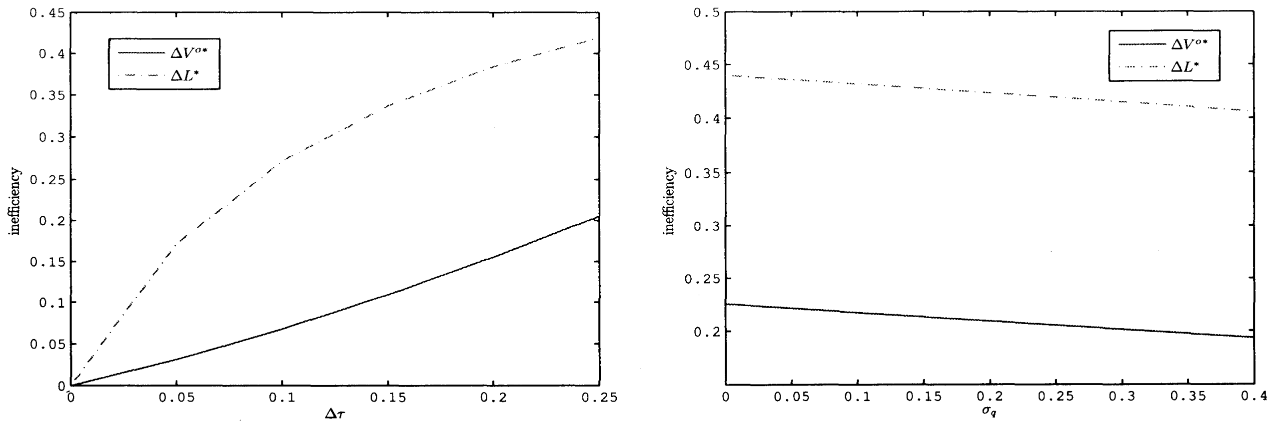

Figure 2: Inefficiency with tax rate differentials and FX rate uncertainty.

(v) the correlation between FX rate and foreign market increases (bottom panel).[5] The former three results are intuitvely right. For example, if the domestic currency is Janpanese yen, the third result implies that investment advances when the Janpanese yen is strong. The latter two results are consistent with the empirical findings in Goldberg and Kolstad (1995), who point that investment advances as FX rate uncertainty rises and becomes more correlated with the uncertainty in foreign market. On the other hand, the default threshold increases with tax rate differentials (top left panel) and debt shifting cost (top right panel). The reason is that, as tax rate differentials increase and debt shifting cost decreases, the coupon level increases, and so does the default threshold. The effect of other three parameters (Δr, ρ, and σ_q) on the default threshold works in the same direction with that on the investment threshold.

5 Conclusions

In this paper, we applied a a first-hitting-time approach via change of mesure to examine the impact of international debt shifting on financing and investment decisions of foreign subsidiary, incorporating FX rate uncertainty. We found that debt shifting induces earlier investment and default of foreign subsidiary, larger coupon level of debt, and higher leverage and credit spread. The quantitative effects of international debt shifting cannot be ignored. Our results are consistent with several empirical findings. When tax rate differentials increase, the optimal leverage of foreign subsidiary at its establishment increases, as Huizinga *et al.* (2008) report. Also, investment of foreign subsidiary advances as FX rate uncertainty rises and becomes more correlated with the uncertainty in foreign market, which echoes the results in Goldberg and Kolstad (1995). Moreover, igonoring FX rate uncertianty and the possibility of debt shifting reduces our model to Sundaresan and Wang (2007).

[5]We set $\rho = 0.2$ in the bottom panel of Figure 3 and Figure 2 to ensure the condition $r_f > \mu_x + \rho\sigma_q\sigma_x$ is satisfied for various σ_q.

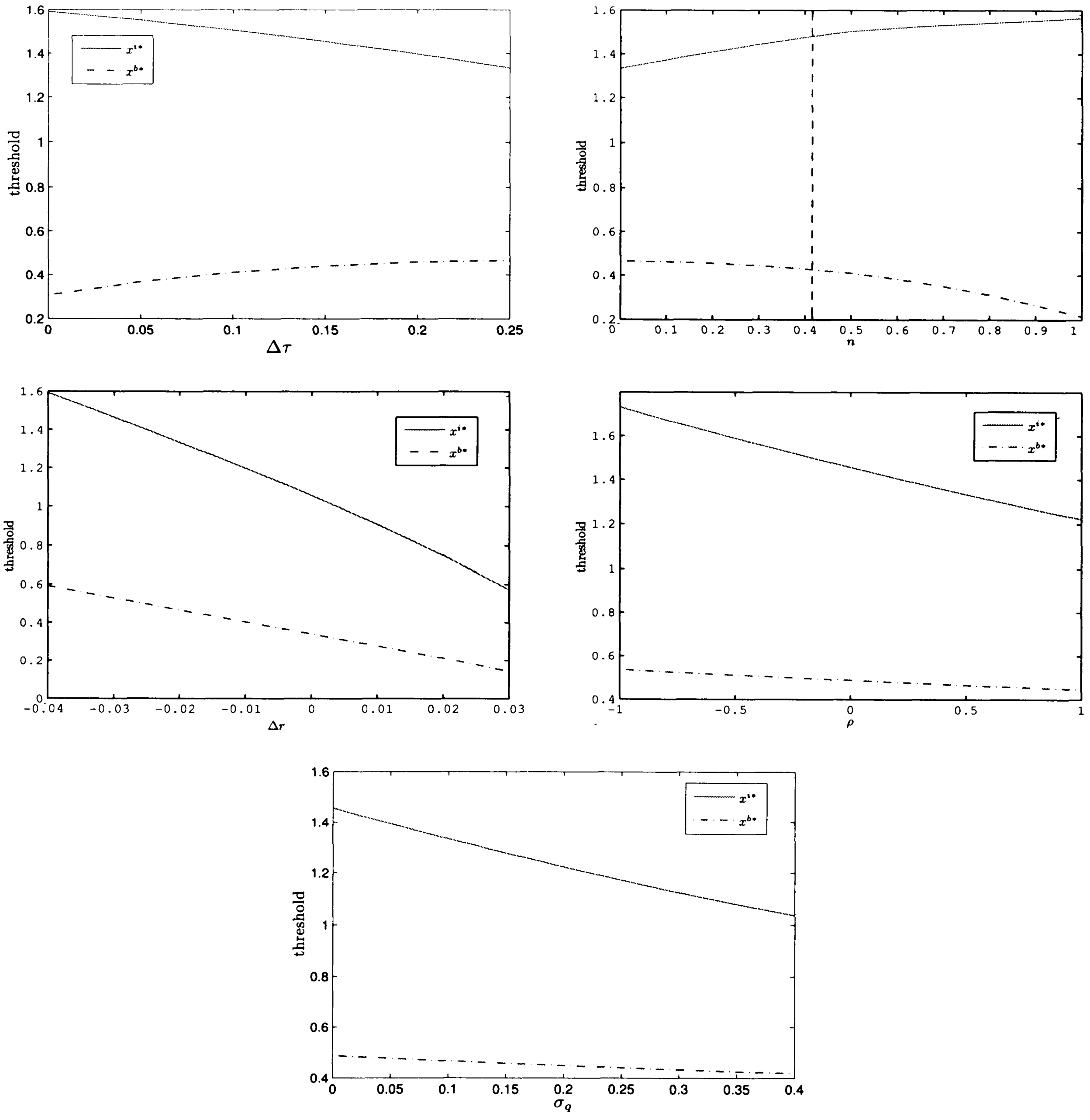

Figure 3: Thresholds of foreign subsidiary with different parameters.

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